1. 2 is a ‘critical value’, e.g. used in solution, or $x = 2$ seen as an asymptote

$x^2 = 2x^2 - 4x \Rightarrow x^2 - 4x = 0$

$x = 0, \quad x = 4$ M1: two other critical values

$x < 0$ B1

$2 < x < 4$ M1: An inequality using the critical value 2 M1 A1 6

[6]

First M mark can be implied by the two correct values, but otherwise a Method must be seen.
≤ appearing: maximum 1 mark penalty (at first occurrence).

2. (a) $m^2 + 2m + 5 = 0 \quad \Rightarrow \quad = -1 \pm 2\mathrm{i}$ M1 A1

$x = e^{-t}(A \cos 2t + B \sin 2t)$
M: Correct form (needs the two different constants) M1 A1 4

(b) $(1, 0) \quad \Rightarrow \quad A = 1$ dB1

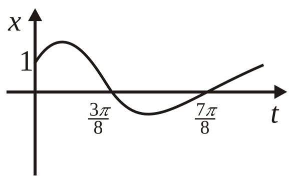
$\dot{x} = -\mathrm{e}^{-t}(A \cos 2t + B \sin 2t) + \mathrm{e}^{-t}(-2A \sin 2t + 2B \cos 2t)$
M: Product diff. attempt dM1

With $A = 1$, $\mathrm{e}^{-t}\{\cos 2t(-1 + 2B) + \sin 2t(-B - 2)\}$

$\dot{x} = 1,\ t = 0 \quad \Rightarrow \quad 1 = -A + 2B$ M1

$B = 1 \qquad (x = \mathrm{e}^{-t}(\cos 2t + \sin 2t))$
M: Use value of A to find B. dM1 A1cso 5

(c)



‘Single oscillation’ between 0 and π B1

Decreasing amplitude (dep. on a turning point) B1ft

Initially increasing to maximum B1ft

Any <u>one</u> correct intercept, whether in terms of π or not: 1 or $\frac{3\pi}{8}$ or $\frac{7\pi}{8}$ B1 4

(Allow degrees: 67.5° or 157.5°) (Allow awrt 0.32π or 1.18 or 2.75)

[13]

(a) First M: Form and attempt to solve auxiliary equation.

2nd M: $Ae^{(-1+2i)\,t} + 5e^{(-1+2i)\,t}$ scores M1, as does $Ae^{m_1 t} + Be^{m_2 t}$ for real m_1, m_2.

(b) B mark and first and third M marks are dependent on the M's in part (a).

(c) First B1: Starts on positive x-axis, dips below t-axis, above t-axis at $t = \pi$, and no more than 2 turning points between 0 and π
(Assume 0 to π if axis is not labelled).

Second B1ft: Increasing amplitude for positive real part of m.
Third B 1ft: Initially decreasing to minimum for negative B.
Initially at maximum for $B = 0$.
Final B1: Dependent on a sketch attempt.

<u>Confusion of variables</u>: Can lose the final A mark in (a).

3. (a)

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$		B1	
$v + x\dfrac{dv}{dx} = \dfrac{3x - 4vx}{4x + 3vx}$	(all in terms of v and x)	M1	
$x\dfrac{dv}{dx} = \dfrac{3 - 4v - v(4 + 3v)}{4 + 3v}$			
(Requires $x\dfrac{dv}{dx}$ = f(v), 2 terms over common denom.)		M1	
$x\dfrac{dv}{dx} = \dfrac{3v^2 + 8v - 3}{3v + 4}$		A1 cso	4

(b)

$\dfrac{3v + 4}{3v^2 + 8v - 3}\,dv = -\dfrac{1}{x}\,dx$	Separating variables	M1	
$\pm \ln x$		B1	
$\dfrac{1}{2}\ln(3v^2 + 8v - 3)$	M: $k\ln(3v^2 + 8v - 3)$	M1 A1	
$\dfrac{1}{2}\ln\left(\dfrac{3y^2}{x^2} + \dfrac{8y}{x} - 3\right) = -\ln x + C$	Or any equivalent form	A1	5

(c) $\frac{3y^2}{x^2} + \frac{8y}{x} - 3 = \frac{A}{x^2}$

Removing ln's correctly at any stage, dep. on having *C*. M1

Using (1, 7) to form an equation in *A* (need not be *A* = ...) M1

(1,7) $\Rightarrow$ $3 \times 49 + 56 - 3 = A$ $\Rightarrow$ $A = 200$ (or equiv., can still be ln)A1

$3y^2 + 8yx - 3x^2 = 200$

$(3y - x)(y + 3x) = 200$ (M dependent on the 2 previous M's) M1 A1 cso 5

[14]

Parts (b) and (c) may well merge.

(b) Partial fractions may be used $\left(A = \frac{3}{2}, B = \frac{1}{2}\right)$, giving $\frac{1}{2}\ln(3v - 1) + \frac{1}{2}\ln(v + 3)$.

(c) Final M requires formation and factorisation of the quadratic.

4. (a) (i) $r^2 \sin^2\theta = a^2 \cos 2\theta \sin^2\theta = a^2(1 - 2\sin^2\theta)\sin^2\theta$ B1 1

$(= a^2(\sin^2\theta - 2\sin^4\theta))$

(ii) $\frac{\mathrm{d}}{\mathrm{d}\theta}(a^2(\sin^2\theta - 2\sin^4\theta)) = a^2(2\sin\theta\cos\theta - 8\sin^2\theta\cos\theta)$, $= 0$

M1, A1, M1

$2 = 8\sin^2\theta$ (Proceed to a $\sin^2\theta = b$) M1

$\sin\theta = \frac{1}{2}$ $\Rightarrow$ $\theta = \frac{\pi}{6}$, $r = \frac{a}{\sqrt{2}}$ A1, A1 cso 6

(b) $\frac{a^2}{2}\int \cos 2\theta \,\mathrm{d}\theta = \frac{a^2}{4}\sin 2\theta$ M: Attempt $\frac{1}{2}\int r^2 \mathrm{d}\theta$, to get $k\sin 2\theta$ M1 A1

$[...]_{\pi/6}^{\pi/4} = \frac{a^2}{4}\left[1 - \frac{\sqrt{3}}{2}\right]$ M: Using correct limits M1 A1

$\Delta = \frac{1}{2}\left(\frac{a}{\sqrt{2}}.\frac{1}{2}\right) \times \left(\frac{a}{\sqrt{2}} - \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}a^2}{16}$

M: Full method for rectangle or triangle M1 A1

$R = \frac{\sqrt{3}a^2}{16} - \frac{a^2}{4}\left[1 - \frac{\sqrt{3}}{2}\right] = \frac{a^2}{16}$ $(3\sqrt{3} - 4)$

M: Subtracting, either way round dM1 A1 cso 8

[15]

(a) (ii) First A1: Correct derivative of a correct expression for $r^2\sin^2\theta$ or $r \sin \theta$.

(b) Final M mark is dependent on the first and third M's.

Attempts at the triangle area by integration: a full method is required for M1.

<u>Missing *a* factors</u>: (or a^2) Maximum one mark penalty in the question.

5. $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$ B1

$\cos \frac{\pi}{10} + i \sin \frac{\pi}{10}$ B1

$\cos\left(\frac{(4k+1)\pi}{10}\right) + i\sin\left(\frac{(4k+1)\pi}{10}\right), k = 2,3,4\text{(or equiv.)}$ M1 A2, 1, 0 5

[$\cos\left(\frac{9\pi}{10}\right) + i\sin\left(\frac{9\pi}{10}\right)$, $\cos\left(\frac{13\pi}{10}\right) + i\sin\left(\frac{13\pi}{10}\right)$, $\cos\left(\frac{17\pi}{10}\right) + i\sin\left(\frac{17\pi}{10}\right)$

[Degrees : 18, 90, 162, 234, 306]

[5]

6. $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_{-1}}{2h} \Rightarrow 2 \approx \frac{y_1 - y_{-1}}{0.2} \Rightarrow y_1 - y_{-1} \approx 0.4$ M1 A1

$\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{2h} \Rightarrow 8 \approx \frac{y_1 - 2y_0 + y_{-1}}{0.01}$

[For M1, an attempt at evaluating $\left(\frac{d^2y}{dx^2}\right)_0$ is required.]

$\Rightarrow \quad y_1 + y_{-1} \approx 2.08$ A1

Subtracting to give $y_{-1} \approx 0.84$ M1 A1 6

[6]

7. (a) Correct method for producing 2nd order differential equation M1

e.g. $\frac{d}{dx}\left\{(1+2x)\frac{dy}{dx}\right\} = \frac{d}{dx}\left\{x + 4y^2\right\}$ attempted

$(1 + 2x)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 1 + 8y\frac{dy}{dx}$ seen + conclusion AG A1 2

(b) Differentiating again w.r.t. x:

$(1 + 2x)\dfrac{d^3y}{dx^3} + 2\dfrac{d^2y}{dx^2} = 8y\dfrac{d^2y}{dx^2} + 8\left(\dfrac{dy}{dx}\right)^2 - 2\dfrac{d^2y}{dx^2}$ or equiv. M1 A2, 1, 0 3

[e.g. $(1 + 2x)\dfrac{d^3y}{dx^3} = 8\left(\dfrac{dy}{dx}\right)^2 + 4(2y-1)\dfrac{d^2y}{dx^2}$

(c) $\dfrac{dy}{dx}$ (at $x = 0$) = 1 B1

Finding $\dfrac{d^2y}{dx^2}$ (at $x = 0$) (= 3) M1

Finding $\dfrac{d^3y}{dx^3}$, at $x = 0$; = 8 [A1 f.t. is on part (c) values only] M1 A1ft

$y = \dfrac{1}{2} + x + \dfrac{3}{2}x^2 + \dfrac{4}{3}x^3 + \ldots$ M1 A1 6

[11]

[Alternative (c):

Polynomial for y: $y = ½ + ax + bx^2 + cx^3 + \ldots$ M1

In given d.e.:

$(1 + 2x)(a + 2bx + 3cx^2 + \ldots) \equiv x + 4(½ + ax + bx^2 + cx^3 + \ldots)^2$ M1A1

a = 1 B1, Complete method for other coefficients M1, answer A1

8. (a) Relating lines and angle (generous) M1

[*angle between* ± 2i *to P and* ± 2 *to P*] A1

Angle between correct lines is $\dfrac{\pi}{2}$ M1 A1 4

Circle
Selecting correct ("top half") semi-circle.

[If algebraic approach:

Method for finding Cartesian equation M1
Correct equation, any form, $\Rightarrow x(x + 2) + y(y - 2) = 0$ A1

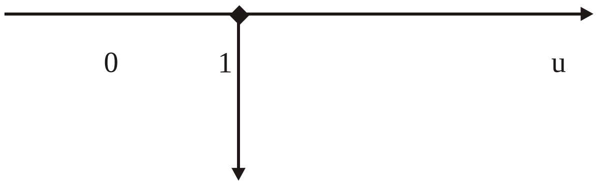
Sketch: showing circle M1
Correct circle { centre (–1, 1)}, choosing only "top half" A1]

(b) |z + 1– i| is radius; = √2 M1 A1 2

(c) $z = \dfrac{2(1+i) - 2\omega}{\omega}$ $\left(= \dfrac{2(1+i)}{\omega} - 2 \right)$ M1

$\dfrac{z - 2i}{z + 2} = \dfrac{2(1+i) - 2(1+i)\omega}{2(1+i)}$ $(= \; - \omega)$ M1 A1

$\text{Arg}\,(1 - \omega) = \dfrac{\pi}{2}$ is line segment, passing through (1,0) A1, A1



A1 6

[12]

Alt (c): $u + iv = \dfrac{2 + 2i}{(x+2) + iy} = \dfrac{(2x + 2y + 4) + i(x + 2 - y)}{(x+2)^2 + y^2}$ M1

$x = -1 + \sqrt{2}\cos\theta,\ y = 1 + \sqrt{2}\sin\theta$ M1

$\Rightarrow w = \dfrac{(2\sqrt{2}\cos\theta + 2\sqrt{2}\sin\theta + 4) + i......}{(2\sqrt{2}\cos\theta + 2\sqrt{2}\sin\theta + 4)}$ $\{= 1 + i\, f(\theta)\}$ A1,

$\Rightarrow$ part of line $u = 1$, show lower "half" of line A1, A1